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Analytical Study of Methods of Functional Analysis and its Applications.

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Abstract :

In the present paper we propose to study of real-valued functions. Functional analysis plays an important role in the applied sciences as well as in mathematics itself. These study is intended to get familiarize with the basic concepts, principles and methods of functional analysis and its applications. We will develop such an abstract scheme for doing calculus in function spaces and other infinite-dimensional spaces, and this is what this paper is about. However, we will look more carefully at the definition of limit in calculus.

Key Words: Functional analysis, Real-valued functions, calculus, Composite Function, L-hospital rule.

Introduction:

A rule **f** that assigns to each member of a nonempty set D a unique member of a set Y is a function from D to Y. We write the relationship between a member x of D and the member y of Y that **f** assigns to x as below,

$$Y=f(x) \qquad \ldots\ldots(1)$$

The set D is the domain of f, denoted by f(x). The members of Y are the possible values of f. If $y_0 \in Y$ and there is an x_0 in D such that $f(x_0) = y_0$ then we say that f attains or assumes the value y_0. The set of values attained by f is the range of **f**.

A real-valued function of a real variable is a function whose domain and range are both subsets of the real. Although we are concerned only with real-valued functions of a real variable in this paper. We will consider situations where the range or domain, or both, are subsets of vector spaces. we will use geometric terminology and intuition in discussing the reals, we will base all proofs on properties and their consequences, not on geometric arguments. Henceforth, we will use the terms real number system and real line synonymously and denote both by the symbol R; also, we will often refer to a real number as a point on the real line

Definitions:

A. Relation:

1. A relation is any correspondence between a set of input values and output values.

2. The set of all inputs (x) is the domain of the relation.

3. The set of all outputs (y or f(x)) is the range of the relation.

B. Function:

1. A function is a special type of relation in which for every member of its domain (x) is associated with exactly one member of its range (y or f(x)).

2. In other words, a function is a relationship in which each input value has a unique out value. For every "x" there is one and only one value of "y" associated with that "x".

3. **Examples:** a. If you are working at Suds Car Wash for an hourly wage, the relationship between the numbers of hours you work and the resulting income you earn is a function.

b. Every holder of a social security card in the United States is assigned a nine-digit social security number.

C. Functional Notation

1. Functional notation is often used to represent functions.

2. *f*(x) is read *f* of x or the value of the function *f* at x.

3. Example: a. If $f(x) = 3x - 2$, then $f(-3) = 3(-3) - 2$; this equals -11, so $f(-3) = -11$

D. Composite Functions

1. Applying one function to the answer of another function is called the Composition of Functions.

2. $f * g(x)$ or $f(g(x))$ is read *f* of *g* of x or the value of the function *f* at the value of the function *g* at x.

E. Inverse Functions:

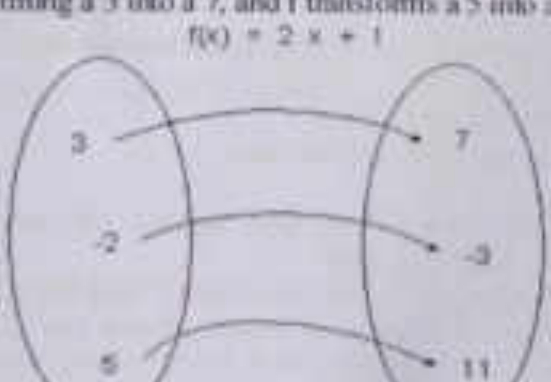
1. If $f(x)$ is a one-to-one function with ordered pairs of the form (x, y), its inverse function, $f^{-1}(x)$, is a one-to-one function with ordered pairs of the form (y,x).

2. To determine the inverse of a linear function, you need to derive the function that "undoes" the original function.

3. Here is a nice way of visualizing what is going on. Since g(f(x)) means "first apply f and then apply g," we can think of this as feeding the output of f into g, and seeing what we get. Here is an illustration of this process.

- **Overview of a Function and Its' Inverse Function**

1. Consider the function f(x) = 2x + 1. This function multiples any number by two and then adds 1. We know how to evaluate f at 3. f(3) = 2*3 + 1 = 7. It helps to think of f as transforming a 3 into a 7, and f transforms a 5 into an 11, etc.



2. Now that we think of f as "acting on" numbers and transforming them, we can define the inverse of f as the function that "undoes" what f did.

3. In other words, the inverse of f needs to take 7 back to 3, and take -3 back to -2, etc. In order to do this the inverse function needs to subtract one to any number and then divide it by 2.

4. Therefore, the Inverse Function would be ☞ g(x) = (x - 1)/2. Then g(7) = 3, g(-3) = -2, and g(11) = 5, so g seems to be undoing what f did, at least for these three values.

5. To prove that g is the inverse of f we must show that this is true for any value of x in the domain of f.

6. In other words, g must take f(x) back to x for all values of x in the domain of f. So, g(f(x)) = x must hold for all x in the domain of f. The way to check this condition is to see that the formula for g(f(x)) simplifies to x.

- **Examples of Functions and their Inverses**

Function	**Inverse Function**
f(x) = -3x	$f^{-1}(x) = \frac{x}{-3}$
f(x) = 5x +2	$f^{-1}(x) = \frac{x-2}{5}$
$g(x) = x^2$	$g^{-1}(x) = \sqrt{x}$

- **How to Find the Inverse Function from the Original Function**

1. Order pairs : If your function is defined as a list of ordered pairs, simply swap the x and y values. Remember, the inverse will be a *function* only if the original function is one-to-one.

2. Reverse the Operations

a. First consider a simple example **f(x) = 3x + 2** The graph of f is a line with slope 3, so it passes the horizontal line test and does have an inverse.

b. There are two steps required to evaluate f at a number x. First we multiply x by 3, then we add 2.

c. Thinking of the inverse function as undoing what f did, we must undo these steps in reverse order.

d. The steps required to evaluate f^{-1} are to first undo the adding of 2 by subtracting 2. Then we undo multiplication by 3 by dividing by 3.

e. Therefore, $f^{-1}(x) = (x - 2)/3$.

3. Algebraically – Steps for Funding the Inverse of a Function – *f*.

a. Replace f(x) by y in the equation describing the function.

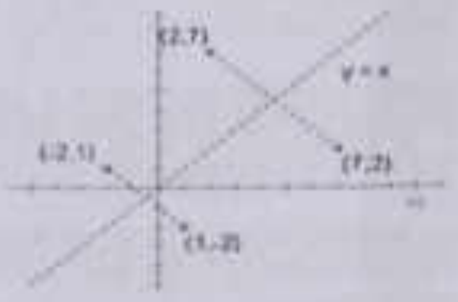
b. Interchange x and y. In other words, replace every x by a y and vice versa.

c. Solve for y.

d. Replace y by $f^{-1}(x)$

- **Graphs of Functions and Their Inverses**

The *graph of an inverse* is the reflection of the original graph over the identity line y=x. The reflection of the point (a,b) about the line y = x is the point (b,a).

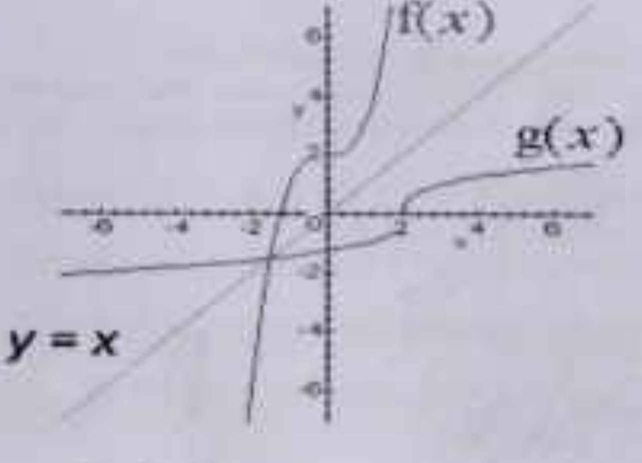


Example:

1. Suppose you have a function, $f(x) = x^3 + 2$. Then $f(2) = 10$ and the point (2,10) is on the graph of f.

2. The inverse of f must take 10 back to 2, i.e. $f^{-1}(10)=2$, so the point (10,2) is on the graph of f^{-1}. The Inverse Function is: $f^{-1}(x) = \sqrt[3]{x-2}$

3. The point (10,2) is the reflection in the line y = x of the point (2,10). The same argument can be made for all points on the graphs of f and f^{-1}. The graph of f^{-1} is the reflection about the line y = x of the graph of f.



Limits and Continuity:

We have seen that the investigation of limits and continuity can be simplified by regarding a given function as the result of addition, subtraction, multiplication, and division of simpler functions. Another operation useful in this connection is composition of functions; that is substitution of one function into another.

We can use these mathematical functions to determine the continuity of the function. There is L-hospital rule, which is very popular in this regard. Further rules like Induction, differentiation, integration can be used to solve the various types of problems from the space.

One of our objectives is to develop rigorously the concepts of limit, continuity, differentiability, and integrability, which you have seen in calculus. To do this requires a better understanding of the real numbers than is provided in calculus.

The purpose of this section is to develop this understanding. Since the utility of the concepts introduced here will not become apparent until we are well into the study of limits and continuity, you should reserve judgment on their value until they are applied. As this occurs, you should reread the applicable parts of this section. This applies especially to the concept of an open covering and to the Heine-Borel and Bolzano-Weierstrass theorems, which will seem mysterious at first.

We assume that you are familiar with the geometric interpretation of the real numbers as points on a line. We will not prove that this interpretation is legitimate, for two reasons:

(1) The proof requires an excursion into the foundations of Euclidean geometry;

(2) Although we will use geometric terminology and intuition in discussing the reals, we will base all proofs on properties and their consequences, not on geometric arguments. Henceforth, we will use the terms real number system and real line synonymously and denote both by the symbol R; also, we will often refer to a real number as a point on the real line.

Conclusion:

Thus the need arises for developing calculus in more general spaces than R^n. Although we have only considered examples, problems requiring calculus in infinite-dimensional spaces arise from many applications and from various disciplines such as economics, engineering, physics, and so on. Mathematicians observed that different problems from varied fields often have related features and properties. This fact was used for an effective unifying approach towards such problems. Hence the advantage of an abstract approach is that it concentrates on the essential facts, so that these facts become clearly visible and one's attention is not disturbed by unimportant details.

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