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A Study of Mathematical Functions in Topological Structure

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Abstract

The aim of the present paper is a kind of revised and enlarged study of the title Variational Convex Analysis, mathematical function, and Duality theory. First we present the basic tools of analysis necessary to develop the core theory and applications. New results concerning duality principles for systems originally modeled by non-linear differential equations. In the context of normal spaces, it is then natural to focus attention on those linear transformations that are also continuous. These are important from the point of view of applications.

Key Words- Convex Analysis, Duality Principal, dynamic matrix, sensor matrix, Laplace transform

1. Introduction

Functional analysis plays an important role in the applied sciences as well as in mathematics itself. These notes are intended to familiarize the student with the basic concepts, principles and methods of functional analysis and its applications, and they are intended for senior undergraduate or beginning graduate students.

The notes are elementary assuming no prerequisites beyond knowledge of linear algebra and ordinary calculus (with ϙ-δ arguments). Measure theory is neither assumed, nor discussed, and no knowledge of topology is required. The notes should hence be accessible to a wide spectrum of students, and may also serve to bridge the gap between linear algebra and advanced functional analysis.

Functional analysis is an abstract branch of mathematics that originated from classical analysis.

A real-valued function of a real variable is a function whose domain and range are both subsets of the real. Although we are concerned only with real-valued functions of a real variable in this section, our definitions are not restricted to this situation. In later sections we will consider situations where the range or domain, or both, are subsets of vector spaces.

We consider continuous maps from a normal space X to a normal space Y . The spaces X and Y have a notion of distance between vectors (namely the norm of the difference between the two vectors). Hence we can talk about continuity of maps between these normal spaces, just as in the case of ordinary calculus.

Since the normal spaces are also vector spaces, linear maps play an important role. Recall that linear maps are those maps that preserve the vector space operations of addition and scalar multiplication. These are already familiar to the reader from elementary linear algebra, and they are called linear transformations.

The set of all bounded linear operators is itself a vector space, with obvious operations of addition and scalar multiplication, and as we shall see, it also has a natural notion of a norm, called the operator norm. Equipped with the operator norm, the vector space of bounded linear operators is a Banach space, provided that the co-domain is a Banach space. This is a useful result, which we will use in order to prove the existence of solutions to integral and differential equations.

2 Definitions

- **State:** A summary of the past information that affects the system's future behavior.
- **State variables:** A minimal set of variables needed to determine the future behavior of the system from the system's inputs.
- **State-space formulation:** A mathematical description of the relationships of the input, output, and the state of the system.

3 Continuous-time systems

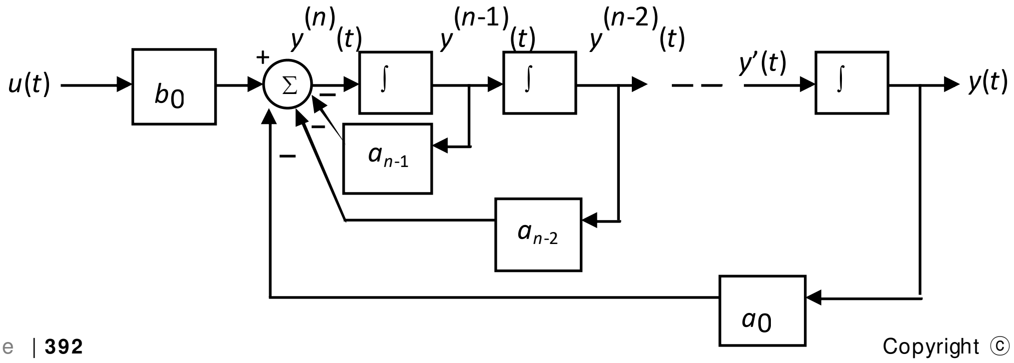
- Differential equation:

$$y^{(n)}(t)+a_{n-1}y^{(n-1)}(t)+\cdots+a_1y'(t)+a_0y(t)=b_0u(t)$$

Solving for the highest order derivative:

$$y^{(n)}(t)=-a_{n-1}y^{(n-1)}(t)-\cdots-a_1y'(t)-a_0y(t)+b_0u(t)$$

- **Block diagram:**



- Converting an n^{th} order differential equation to a set of n first-order differential equations: Let $x_i(t)$ represent the outputs of the integrators. As a result we can choose them as the state variables of the system.

$x_1(t) = y(t)$

$x_2(t) = y'(t) = x_1'(t)$

$x_3(t) = y''(t) = x_2'(t)$

…

$x_n(t) = y^{(n-1)}(t) = x_{n-1}'(t)$

Rewriting these equations:

$x_1'(t) = x_2(t)$

$x_2'(t) = x_3(t)$

…

$x_{n-1}'(t) = x_n(t)$

$x_n'(t) = y^{(n)}(t) = b_0\, u(t) - a_0\, x_1(t) - \ldots - a_{n-1}x_n(t)$

- Matrix formulation:

$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \\ . \\ . \\ . \\ x_{n-1}'(t) \\ x_n'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & . & . & 0 & 0 \\ 0 & 0 & 1 & . & . & 0 & 0 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & 0 & 1 \\ -a_0 & -a_1 & -a_2 & . & . & -a_{n-2} & -a_{n-1} \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \\ . \\ . \\ . \\ x_{n-1}(t) \\ x_n(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 0 \\ b_0 \end{bmatrix} \cdot u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_{n-1}(t) \\ x_n(t) \end{bmatrix} + [0] \cdot u(t)$$

- Vector notation:

  $\underline{x}'(t) = \underline{A}\,\underline{x}(t) + \underline{B}\,u(t)$

  $y(t) = \underline{C}\,\underline{x}(t) + \underline{D}\,u(t)$

  where

  - $\underline{x}(t)$ is the state vector
  - $y(t)$ is the output
  - $u(t)$ is the input
  - $\underline{A}$ is called the state (dynamic) matrix
  - $\underline{B}$ is called the input matrix
  - $\underline{C}$ is called the output (sensor) matrix
  - $\underline{D}$ is called the feed through matrix

  Now, an n^{th} order differential equation has been converted to a set of n first-order differential equations.

- For a multi-input/multi-output system:

  $\underline{x}'(t) = \underline{A}\,\underline{x}(t) + \underline{B}\,\underline{u}(t)$

  $\underline{y}(t) = \underline{C}\,\underline{x}(t) + \underline{D}\,\underline{u}(t)$

  For a system with n state variable, p inputs and q outputs:

  $\underline{A}$ is $n \times n$

  $\underline{B}$ is $n \times p$

  $\underline{C}$ is $q \times n$

  $\underline{D}$ is $q \times p$

  $\underline{x}(t)$ is $n \times 1$

  $\underline{y}(t)$ is $q \times 1$

  $\underline{u}(t)$ is $p \times 1$

- The transfer function matrix:

  Taking the Laplace transform of the state-variable vector equation we obtain:

  $s\,\underline{X}(s) - \underline{x}(0) = \underline{A}\,\underline{X}(s) + \underline{B}\,\underline{U}(s)$

$\Rightarrow \qquad \underline{X}(s) = (s\,\underline{I} - \underline{A})^{-1}\,\{\underline{x}(0) + \underline{B}\,\underline{U}(s)\}$

and $\qquad \underline{Y}(s) = \underline{C}\,\underline{X}(s) + \underline{D}\,\underline{U}(s)$

$\Rightarrow \qquad \underline{Y}(s) = \underline{C}\,(s\,\underline{I} - \underline{A})^{-1}\,\{\underline{x}(0) + \underline{B}\,\underline{U}(s)\} + \underline{D}\,\underline{U}(s)$

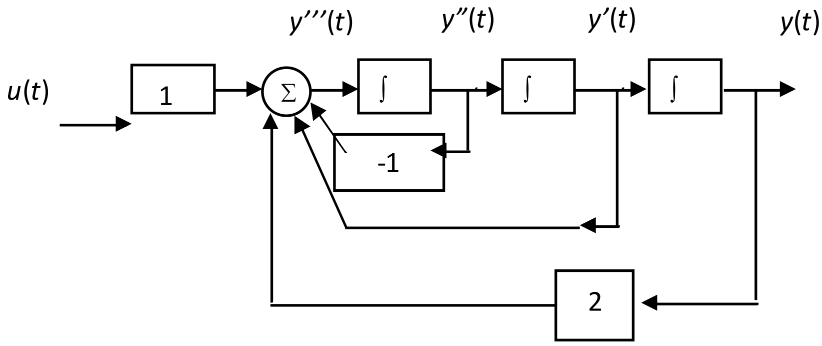
Finally, setting $\underline{x}(0) = \underline{0}$ gives us the transfer function matrix:

$$\underline{Y}(s) = \underline{C}\,(s\,\underline{I} - \underline{A})^{-1}\,\underline{B}\,\underline{U}(s) + \underline{D}\,\underline{U}(s) = \{\underline{C}\,(s\,\underline{I} - \underline{A})^{-1}\,\underline{B} + \underline{D}\}\,\underline{U}(s)$$

or $\qquad \underline{G}(s) = \underline{C}\,(s\,\underline{I} - \underline{A})^{-1}\,\underline{B} + \underline{D}$

- Example 1:

$y'''(t) + y''(t) - 2y'(t) + 3y(t) = u(t)$



$$\begin{bmatrix} x_1'(t) \\ x_2'(t) \\ x_3'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} \cdot u(t)$$

  - Example 2 :

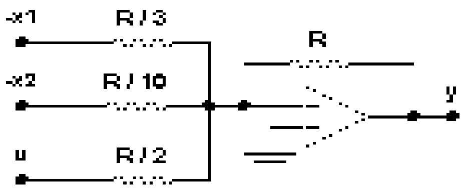
$$\mathbf{x}' = \begin{bmatrix} -2 & 4 \\ 0 & 5 \end{bmatrix} \cdot \mathbf{x} + \begin{bmatrix} 2 \\ -4 \end{bmatrix} \cdot \mathbf{u}$$

$$y = \begin{bmatrix} 3 & 10 \end{bmatrix} \cdot \mathbf{x} + \begin{bmatrix} -2 \end{bmatrix} \cdot \mathbf{u}$$

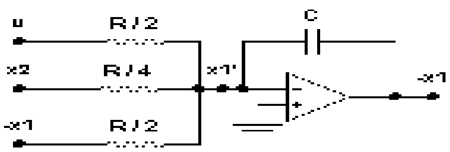
Rearranging the terms we have:

$$-y = -3x_1 - 10x_2 + 2u$$
$$x_1' = -2x_1 + 4x_2 + 2u$$
$$-x_2' = -5x_2 + 4u$$

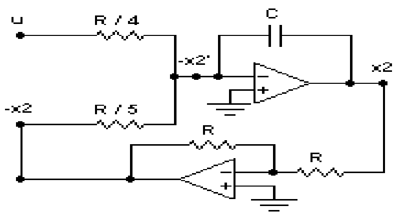
- Constructing *y*:



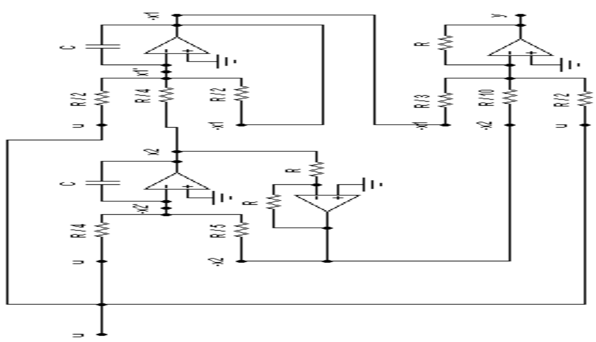
- Constructing x_1:



- Constructing x_2:



- Connecting the stages:



2. Conclusion

The set of all bounded linear operators is itself a vector space, with obvious operations of addition and scalar multiplication, and as we shall see, it also has a natural notion of a norm, called the operator norm. Equipped with the operator norm, the vector space of bounded linear operators is a Banach space, provided that the co-domain is a Banach space. Thus the derivative at a point will be a bounded linear operator. This theme of local approximation is the basis of several computational methods in nonlinear functional analysis.

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