
SOLUTION OF FORCED AND FREE CONVECTION FLOW OF DISSIPATIVE FLUID PAST AN INFINITE VERTICAL PLATE

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ABSTRACT

An approximate solution of forced and free convection flow of dissipative fluid past an infinite vertical Plate, is derived by explicit finite difference technique by taking into account viscous dissipative heat. It is observed that the velocity decreases near the plate and the increases far away from the plate. Greater Viscous dissipative heat causes a rise in the velocity but the velocity decreases with increasing the Prandtl number for large t . An increase in G or t also increases in the skin friction but the rate of heat transfer decreases.

Keywords: *Viscous dissipative heat, Prandtl number, Grashof number, Skin friction.*

INTRODUCTION

Siegel (1958) Schetz and Eichhorn (1962) Menold and Yang (1962) Chung and Anderson (1961), Goldstein and Briggs (1964) and Sugawara and Michiyoshi (1951) Soundalgekar, Lahurikar and Pohnerkar (1997) studied the unsteady free convection flow under various conditions past an infinite vertical plate. Goldstein and Eckert (1960), confirmed experimentally some of these theoretical predictions. In all these studies, the infinite plate was assumed to be stationary and the fluid was supposed to move due to temperature difference only. If the fluid is stationary and the infinite plate surrounded by stationary fluid is given an impulsive motion along with its temperature raised to such that, where is the temperature of the surrounding fluid how the shape of fluid flowing takes its shape? This was studied by Soundalgekar (1977) in case of an isothermal plate. The effect of free convection currents on the flow and the skin friction were studied in this paper.

Combined free and forced convection flow past a semi-infinite vertical plate was first studied by Acrivos (1958), Kliegel (1959) who solved the equations by using the Karman-Pohlhausen method. However another physical situation which is often experienced in the industrial application is the unsteady free and forced convective flow past an infinite vertical isothermal plate of an incompressible fluid. This situation studied by Jahagirdar and Lahurikar (1989) without considering the dissipative heat.

In some of these papers the effect of viscous dissipative heat was assumed to be neglected. Gebhart (1962) has studied and get the result that when the temperature difference is small or in high Prandtl number fluids or when the gravitational field is of high intensity, viscous dissipative heat should be taken into account in steady free convection flow past a semi-infinite vertical plate. Following this assumption Soundalgekar, Bhat and Mohiuddin (1979) studied the effect of free convection currents on the flow past impulsively started infinite plate, in this case the problem is governed by a coupled non-linear system of partial differential equations This problem was solved by finite difference technique.

It has been proposed to study forced and free convection flow of dissipative fluid past an infinite vertical Plate. As the problem is governed by coupled nonlinear system of partial difference equations exact solutions are not possible, so we employ explicit finite difference method.

MATHEMATICAL ANALYSIS :

Here we consider the unsteady free and forced convection flow of a viscous incompressible fluid past an infinite vertical isothermal plate in the upward direction in presence of dissipative heat. The x -axis is taken along the plate in the vertically upward direction and the y -axis taken normal to the plate. Initially at both the plate and the fluid are stationary and at the same temperature. At time $t=0$ the plate temperature is raised to T_w and the fluid starts moving upward with velocity U_0 . Then the difference between the plate temperature and the ambient temperature causes the free convection currents to flow near the plate modifying the fluid flow. The physical variables are functions of x and t only. Then under usual Boussinesq's approximation, by the following system of coupled partial differential equation in non dimensional form

$$\frac{\partial u}{\partial t} = G\theta + \frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + Pr E \left(\frac{\partial u}{\partial y} \right)^2 \tag{2}$$

with following initial and boundary conditions.

$$\begin{aligned} u = 0, \quad \theta = 0 & \quad \text{for all } y, t \leq 0 \\ u = 0, \quad \theta = 1 & \quad \text{at } y = 0 \quad t > 0 \\ u = 1 \quad \theta = 0 & \quad \text{as } y \rightarrow \infty, \quad t > 0 \end{aligned} \tag{3}$$

On introducing following non dimensional quantities

$$\begin{aligned} U = \frac{u'}{U_0} \quad t = \frac{t' U_0^2}{\nu} \quad y = \frac{y' U_0}{\nu} \quad Pr = \frac{\mu C_p}{k} \\ G = \frac{\nu g \beta (T'_w - T'_\infty)}{U_0^3}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, E = \frac{U_0^2}{C_p (T'_w - T'_\infty)} \end{aligned} \tag{4}$$

Here G is the Grashof number Pr is the Prandtl number and E is the Eckert number. These are coupled non-linear equations, which have no exact solution or approximate solution. Hence we solve it by explicit finite difference method. These equations reduce the following form.

$$\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = G\theta_{i,j} + \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{(\Delta y)^2} \tag{5}$$

$$Pr \frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t} = \frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{(\Delta y)^2} + \left(\frac{u_{i+1,j} - u_{i,j}}{\Delta y} \right)^2 \tag{6}$$

With following initial and boundary conditions in finite difference form

$$\begin{aligned} u(i, 0) = 0, \quad \theta(t, 0) = 0 & \quad \text{for all } i \text{ except } i = 0 \\ u(0, 0) = 0, \quad \theta(0, 0) = 1 \\ u(0, j) = 0, \quad \theta(0, j) = 1 & \quad \text{for all } j \\ u(35, j) = 1, \quad \theta(35, j) = 0 & \quad y \rightarrow \infty = 35 \end{aligned} \tag{7}$$

Here i correspond to y and j corresponds to t. Here infinity is taken as y=3.5 because from the exact solution of equations (1) & (2) and for E=0 it has been observed that u and tends to 1 and zero around y= 3.5 for all value of Pr. velocities and temperatures is computed from equation (5) to (7) The procedure is repeated till t=1 i.e.,j=400 During computation was chosen as 0.00125. These conditions were carried for Pr=0.71, 7 and E=0.1,0.2 and 0.4. To judge the accuracy of the convergence and stability of the finite difference scheme, the same program was run with smaller values of t i.e.t =0.0009 and 0.001 and no significant change was observed. Hence we conclude that the finite difference scheme is stable and convergent.

We calculate the skin-friction and rate of heat transfer defined by following non dimensional quantities by using the five point Newton Coates formula and entered in the table.

$$\tau = \left(\frac{\tau'}{\rho U_0^2} \right) = \left(\frac{-du}{dy} \right)_{y=0}$$

$$q = \left(\frac{vq'}{kU_0 \Delta T} \right) = - \left(\frac{d\theta}{dy} \right)_{y=0} \tag{8}$$

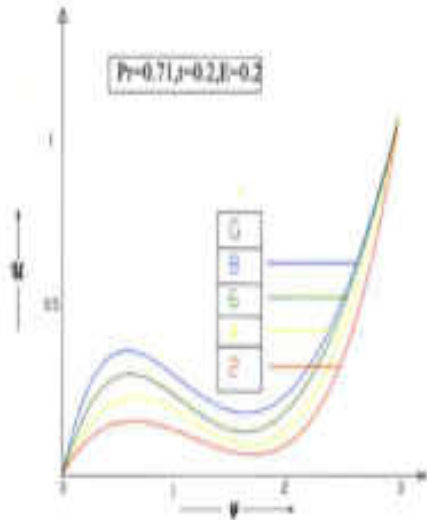


Fig. 1. Velocity profile

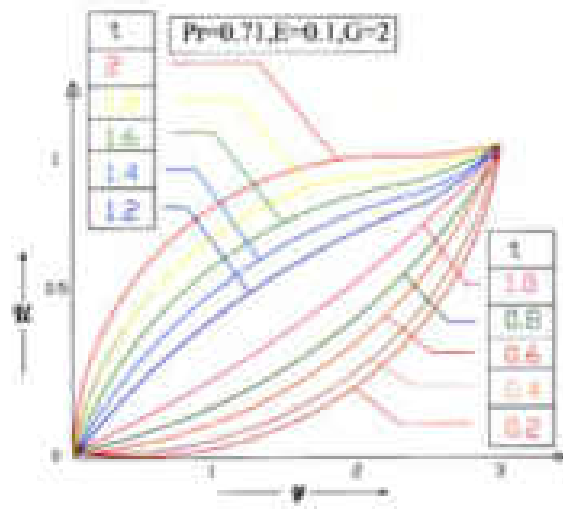


Fig. 2. Velocity profile

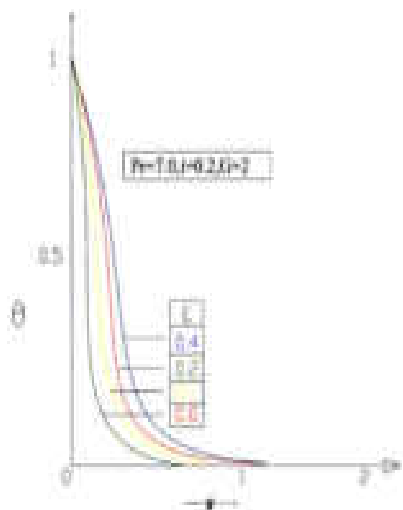


Fig-3: Temperature profile

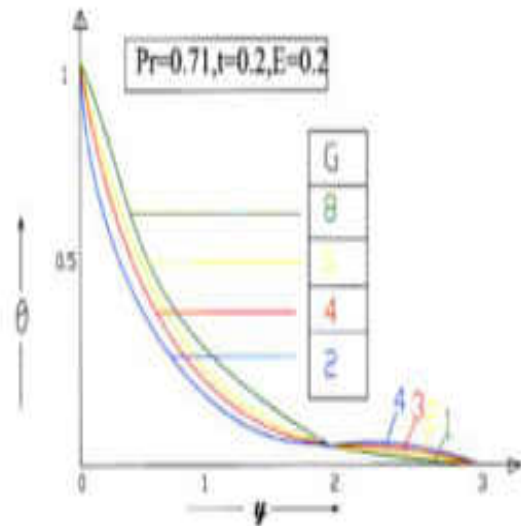


Fig-4: Temperature profile

t	E	G	Pr= 0.71		Pr= 7	
			-τ	-q	-τ	-q
0.2	0.0	2	0.548661	1.057851	0.275831	3.343185
	0.1	2	0.548696	1.056722	0.275836	3.342900
	0.4		0.548801	1.053334	0.275851	3.342067
0.2	0.2	4	1.097791	1.049022	0.551706	3.340955
		6	1.647578	1.038052	0.827688	3.338174
0.4	0.1	2	0.782358	0.744987	0.397351	2.355254
0.6			0.986162	0.601883	0.513918	1.919075

TABLE (I)

RESULT AND CONCLUSIONS

From fig (1) to (4) and Table (I) it is observed that

1. Greater Viscous dissipative heat causes a rise in the velocity but the velocity decreases with increasing the Prandtl number for large t . When Prandtl number increases the flow is unstable for small t .
2. The Grashof number increases the velocity also increases and flow become unstable.
3. An increase in t leads to an increase in the velocity.
4. Greater viscous dissipative heat causes a rise in the skin -friction but an increase in Pr leads to a decrease in the skin-friction. An increase in G or t also increases in the skin -friction.
5. The rate of heat transfer is found to decrease with increasing G, E or t .
6. Greater viscous dissipative heat causes a rise in the skin friction but an increase in Pr leads to a decrease in the skin-friction.
7. Increase in the Grashof number G or in t , skin -friction also increases.

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