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FUZZIFICATION OF LINEAR SPACES

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ABSTRACT
Analyze the concept of fuzzy linear spaces (FLS) and we have proposed the redefined notion of fuzzy linear spaces and have established that the proposed definition is more general and appealing than that of Nanda and Biswas. The notion of product (*) of two fuzzy linear spaces has been proposed and it has been observed that the product is again a fuzzy linear space under the new definition. In other words, we can say that these structures are preserved under the product (*). We observe that it is more general than its classical counter part.

1. INTRODUCTION

The concept of fuzzy linear spaces was introduced by ***Sudarsan Nanda*** in 1986 and was again redefined by **Biswas** in 1989. It is expected that several results from linear algebra and functional analysis can be extended to the concept of fuzzy setting. ***Nanda*** propounded the notion of fuzzy linear spaces in a linear space as follows:

2. FUZZY LINEAR SPACE

Let F be a fuzzy field in a field (X, + , .) with membership function F(λ). Let Y be a linear space over F and V be a fuzzy subset of Y with membership function V(x). Then, V is called as a fuzzy linear space in Y if the following postulates are satisfied:

(i) $V(x + y) \geq \min\{V(x),V(y)\}, \forall\ x , y \in Y$

(ii) $V(\lambda x) \geq \min\{F(\lambda),V(x)\}, \forall\ \lambda \in F$ and $\forall\ x \in Y$

(iii) $V(0) = 1$

In case F is an ordinary field then, $F(\lambda) = 1$ and hence

$V(\lambda x) \geq \min\{1,V(x)\}, \forall \lambda \in F$ and $x \in Y$

$= V(x)$

Hence, for F to be an ordinary field, the (ii) postulate may be considered as

$V(\lambda x) \geq V(x), \forall\ \lambda \in F$ and $x \in Y$

Now we will analyze the definition of fuzzy linear space introduced by ***Nanda***.

Let us consider the case when F and V both are classical set. Then , we have $F(\lambda) = 1$, $V(x) = 1$ and $V(y) = 1$ for all $x , y \in F$

and $\lambda \in F$.

Hence, from condition (i), we have

$V(x + y) = 1 \Rightarrow x + y \in V$

Thus, we get that $x , y \in V \Rightarrow x + y \in V$.

Further, from condition (ii), we get

$V(\lambda x) \geq \min\{1,1\} = 1$

i.e. $V(\lambda x) = 1 \Rightarrow \lambda x \in V$. That is, $x \in V, \lambda \in F \Rightarrow \lambda x \in V$.

It follows that V is closed under addition and scalar multiplication.

Thus, on the basis of above discussion we arrive at the conclusion that the definition of fuzzy linear space has been considered in such a way that when F and V both are considered as an ordinary subset, V turns out to be a subspace of Y.

Alternatively, For all $x , y \in Y$. and $\lambda , \mu \in F$, we have

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$V(\lambda x + \mu y) \geq \min\{V(\lambda x), V(\mu y)\}$

$\geq \min\{\min\{F(\lambda), V(x)\}, \min\{F(\mu), V(y)\}\}$---(1)

If V and F both are ordinary subset then, we get

$V(x) = 1 = V(y)$ and $F(\lambda) = 1 = F(\mu)$

i.e. $V(\lambda x + \mu y) \geq \min\{\min\{1,1\}, \min\{1,1\}\} = 1$

i.e. $V(\lambda x + \mu y) = 1 \Rightarrow \lambda x + \mu y \in V$

That is for all $\lambda, \mu \in F$ and $x, y \in V \Rightarrow \lambda x + \mu y \in V$ .It follows that V is a subspace of Y. So, again we are in a position to say that the notion of fuzzy linear space has been considered in such a way that when F and V both are taken to be an ordinary subset under above definition of fuzzy linear space V turns out to be a subspace of Y.

In case when F is an ordinary field in X, then $F(\mu) = F(\lambda) = 1$. And hence in this case condition (1) reduces to

$V(\lambda x + \mu y) \geq \min\{\min\{1, V(x)\}, \min\{1, V(y)\}\}$

$= \min\{V(x), V(y)\}$

i.e. $V(\lambda x + \mu y) \geq \min\{V(x), V(y)\}$

The concept of fuzzy linear space was redefined by ***R.Biswas*** in 1989 as he was not satisfied by the definition of fuzzy linear space proposed by ***Nanda***. ***Biswas*** redefined the notion of fuzzy linear space, as follows:

3. REDEFINED FUZZY LINEAR SPACE

Let F be a fuzzy field in a field (X, +, .) with membership function $F(\lambda)$. Let Y be a linear space over F and V is a fuzzy subset of Y with membership function $V(x)$. Then, V is said to be a fuzzy linear space in Y if the following conditions are satisfied:

(i) $V(x + y) \geq \min\{V(x), V(y), \forall x, y \in Y$

(ii) $V(\lambda x) \geq \min\{F(\lambda), V(x)\}, \forall \lambda \in F, x \in Y$

Now our aim is to find out the reason for deleting the (iii) condition in the definition of fuzzy linear space proposed by ***Biswas***. In our opinion the reason behind this is the proposition 4.5 in the paper of ***Nanda***. It states that- V is a fuzzy linear space in Y(over a fuzzy field F in X) if and only if

$V(\lambda x + \mu y) \geq \min\{\min\{F(\lambda), V(x)\}, \min\{F(\mu), V(y)\}\}$---(1)

for all $\lambda, \mu \in F$ and $x, y \in Y$

On putting $\lambda = \mu = 1$, we get

$V(x + y) = V(1x+1y) \geq \min\{\min\{F(1), V(x)\}, \min\{F(1), V(y)\}\}$

$= \min\{\min\{1, V(x)\}, \min\{1, V(y)\}\}$

$= \min\{V(x), V(y)\}$

i.e $V(x + y) \geq \min\{V(x), V(y)\}, \forall x, y \in Y$

Again on putting $\mu = 0$ in (1), we get

$V(\lambda x + 0y) = V(\lambda x + 0) \geq \min\{V(\lambda x), V(0)\}$

$= \min\{V(\lambda x), 1\}$

$= V(\lambda x)$

$\geq \min\{F(\lambda), V(x)\}$

i.e $V(\lambda x) \geq \min\{F(\lambda), V(x)\}, \forall \lambda \in F, x \in Y$ ---(2)

Further, if F is an ordinary field in X, then $F(\lambda) = 1$

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Now, from equation (2), we get

$V(\lambda x) \geq \min \{1, V(x)\}$

$= V(x)$

i.e $\quad V(\lambda x) \geq V(x), \forall \lambda \in F = X$ and $x \in Y$

Also, $V(0) = V(x-x) = V(1x - 1x)$

$\geq \min\{V(1x), V(-1x)\}$

$\geq \min\{\min\{F(1), V(x)\}, V(x)\}$

$= \min \{V(x), V(x)\}$

$= V(x)$

i.e $\quad V(0) \geq V(x)$

Thus, we observe that the (iii) condition in the definition of ***Nanda*** is not deducible from the proposition 4.5 and so it has been dropped.

We may like to redefine the concept of fuzzy linear spaces in the following way:

4. IMPROVED DEFINITION OF FUZZY LINEAR SPACE

Let F be a fuzzy field in a field (X, +, .). Let Y be a linear space over F and V is a fuzzy subset of Y. Then V be called as a fuzzy linear space in Y if the following condition are satisfied:

(i) $V(x + y) \geq V(x). V(y), \forall x, y \in Y$

(ii) $V(\lambda x) \geq F(\lambda .V(x), \forall \lambda \in F$ and $x \in Y$

If F is an ordinary field (in particular if F = X), then F $(\lambda) = 1$. Then the, (ii) condition reduces to

$V(\lambda x) \geq V(x), \forall \lambda \in Y$

Now we claim that our definition of fuzzy linear space is more general than that of ***Biswas***.

For, if V is a fuzzy linear space in Y in the sense of ***Biswas***, then

$V(x + y) \geq \min\{V(x), V(y)\} \geq V(x).V(y), \forall x, y \in Y$

and $\quad V(\lambda x) \geq \min\{ F(\lambda), V(x)\} \geq F(\lambda).F(x), \forall \lambda \in F, x \in Y$

It follows that V is a fuzzy linear space according to our definition when it is a fuzzy linear space according to ***Biswas***.

Next, we suppose that V is a fuzzy linear space in Y according to our definition.

That is $\quad V(x + y) \geq V(x).V(y)$

And $\quad V(\lambda x) \geq F(\lambda).V(x)$

But $\quad V(x).V(y) \geq \min\{V(x), V(y)\}$

and $\quad F(\lambda).V(x) \geq \min\{F(\lambda), V(x)\}$are not true.

Therefore, V is not a fuzzy linear space in view of ***Biswas***, while it is a fuzzy linear space under our definition.

5. MULTIPLICATION OF FUZZY LINEAR SPACES

Let Y be a linear space over a fuzzy field F and U and V are two fuzzy linear spaces in Y. Then, we define multiplication (*) between any two fuzzy linear spaces as follows:

$(U*V)(x) = U(x).V(x) \ \forall x \in Y$

Evidently U*U U

For $(U*U)(x) = U(x).U(x)$

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$= [U(x)]^2$

$\leq U(x), \forall(x) \in Y$

Therefore U*U U

THEOREM

The product of two fuzzy linear spaces in a linear space Y over a fuzzy field F in a field X is again a fuzzy linear space in Y.

PROOF: Let us assume that V_1 and V_2 be any two fuzzy linear spaces in a linear space Y over an ordinary field F in X. Then, for all x, $y \in Y$ and $\lambda \in X$, we have

$V_1(x + y) \geq V_1(x).V_1(y)$ and $V_2(x + y) \geq V_2(x).V_2(y)$

Also $V_1(\lambda x) \geq V_1(x)$ and $V_2(\lambda x) \geq V_2(x)$

Now our aim is to show that the product (*) of V_1 and V_2 i.e. $V_1 * V_2$ is again a fuzzy linear space in Y. The product (*) of fuzzy linear spaces in Y is defined as

$(V_1 * V_2)(x) = V_1(x).V_2(x)$

Then, we have

$(V_1 * V_2)(x + y) = V_1(x + y).V_2(x + y)$

$\geq V_1(x).V_1(y).V_2(x).V_2(y)$

$= V_1(x).V_2(x).V_1(y).V_2(y)$

$= (V_1 * V_2)(x).(V_1 * V_2)(y)$

$\therefore \quad (V_1 * V_2)(x + y) \geq V_1(x + y).(V_1 * V_2)(y)$

Also $\quad (V_1 * V_2)(\lambda x) = V_1(\lambda x).V_2(\lambda x)$

$\geq V_1(x).V_2(x)$

$= (V_1 * V_2)(x)$

i.e. $\quad (V_1 * V_2)(\lambda x) \geq (V_1 * V_2)(x)$

This establishes the claim that the product of linear spaces V_1 and V_2 i.e. $V_1 * V_2$ is also a fuzzy linear space in Y.

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